Clee Hill Community Academy Calculation Policy

Examples of calculation methods for each year group and the progression between each method

Rationale

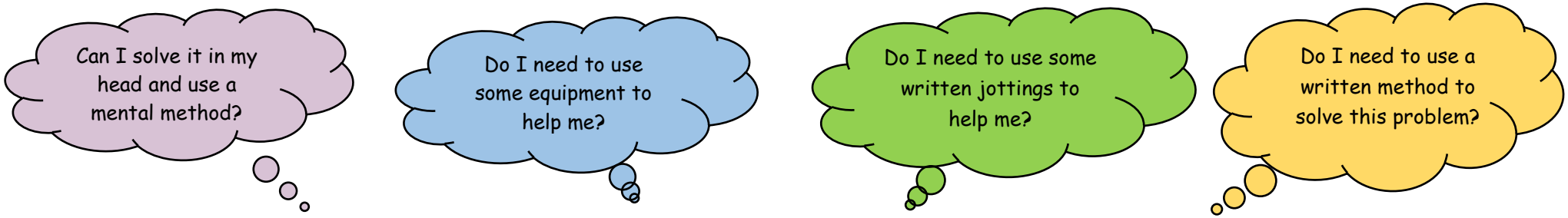
This calculation policy has been created to meet the expectations of the new national curriculum but most importantly the learning needs of our children at the Clee Hill Community Academy. The methods chosen match the national curriculum but have also been specifically selected after consideration of our children's learning styles.

Age Related Expectations

The policy has been organised by year group, considering the national curriculum 2014 expectations. The new curriculum focuses on skills and mastery and is not about moving children on to the next method as soon as they can do the one before. Working and more complex and richer problems rather than new methods will support this 'mastering' of maths. In the Early Years Foundation Stage, we follow the Development Matters progression and the Early Learning Goals.

Mental Methods

Skills in each area of calculation are presented as mental or written skills; however there is a close correlation between them. The written methods in this document are important but as children become more mature and confident with their calculation, they need to also start following these 4 steps when approaching problems:



Interactive policy

Our Academy has adopted the White Rose Hub's calculation document, adapting it to match with our school's approach and interactive links depicted in the videos show some approaches to using resources which is aimed to support new staff and parents.

Reception: Addition

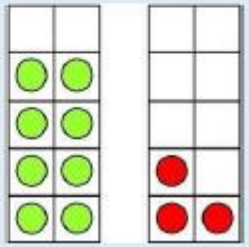
Focus:

In Reception children will be encouraged to develop a fascination with numbers; solving and creating everyday maths problems. They will use mathematical skills as part of real-life activities.

Key vocabulary

add, more, and, make, altogether, double, most, count on, number line

Reception Key skills: Mental Calculation	Reception Key skills: Written Calculation
• Finds one more than a group of up to five and then up to ten objects and then is able to say one more than a given number to 20. • Using quantities and objects they can combine two groups together and give the total. • Using objects they add two single-digit numbers together and can count on to find the answer. • Children understand that five fingers on each hand make a total of ten fingers altogether. • Children understand that two rows of three eggs in the box make six eggs altogether. • Children verbalise the calculations they are doing. • Children start to use the vocabulary of addition. •	• There is no requirement for children to make written recording of their work but children can be encouraged to make their own jottings, drawings to explain what they are doing / have done. • Model ways to record using standard notation when appropriate.

Mental calculation	Numicon  Numicon balance for understanding of equals  Objects and wooden numbers  A full range of different objects to count and combine, natural objects, seeds, sticks, pine cones, straws, counters, dice, lego bricks, people, small animals etc.. Songs and rhymes that add on one more each time. Total of spots on dominoes  Adding cubes  Number tracks the objects can be placed in to count up a total  Bead strings for counting on   Ten frames – how many more do I need to add to make 10?
Informal written recording	 Number sentences modelled  Modelling number sentences next to apparatus

Year 1: Addition

Focus: Adding with 1 digit and 2 digit numbers to 20, including 0.

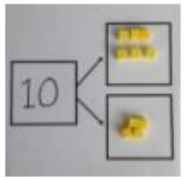
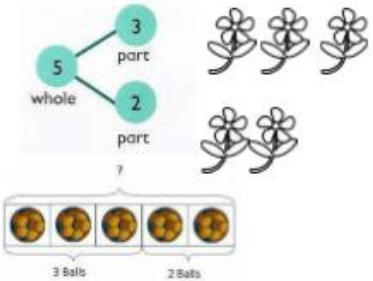
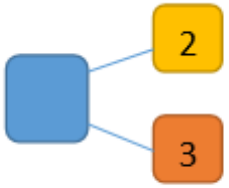
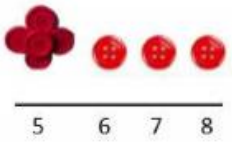
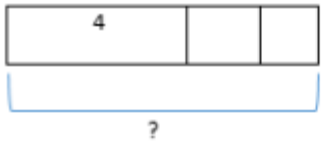
In year 1 the children will build on their knowledge of numbers to 20 from the Foundation Stage. They will begin by using simple strategies to add two group of objects together and move onto recording their number sentences orally and written.

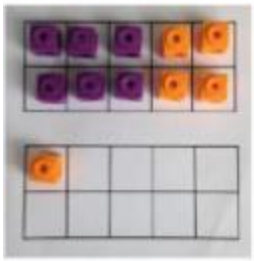
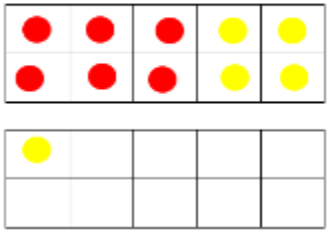
Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Year 1 Key skills: Mental Calculation	Year 1 Key skills: Written Calculation
• Read numbers to 100 in numerals • Count to and across 100 • Recall bonds to 10 and 20, and addition facts within 20 ('story of' 5, 6, 7, 8, 9 and 10) • Count on in ones from a given 2-digit number • Add two single-digit numbers by counting on • Recognise doubles to double 6 • Count on in tens from any given 2-digit number • Add 10 to any given 2-digit number • Add three single-digit numbers spotting doubles or pairs to 10 • Add by putting the larger number first • Use number facts to add single-digit numbers to two-digit numbers, e.g. use 4 + 3 to work out 24 + 3, 34 + 3…	• Write numbers to 100 in numerals • Write numbers 1-20 in words • Calculations recorded as number sentences or simple jumps

Year 1: Addition

Area of addition	Concrete	Pictorial	Abstract	Video link
Number bonds of 5,6,7,8,9,10	   Use cubes to add two numbers together as a group or in a bar. 	 Use pictures to add two numbers together as a group or in a bar. 	2 + 3 = 5 3 + 2 = 5 5 = 3 + 2 5 = 2 + 3  Use the part-part-whole diagram as shown above to move into the abstract.	https://vimeo.com/194167506
Counting	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.   	Use a number line to count on in ones.  A bar model which encourages the children to count on 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? 4 + 2  Written stories with number representation 5 + 3 = 8	https://vimeo.com/194168153

Regrouping to make 10	  6 + 5 = 11 Start with the bigger number and use the smaller number to make 10.	 6 + 5 = 11 4 1 6 + 4 =10 10 + 1 = 11 Children to draw the ten frame and counters/cubes 	Children to develop an understanding of equality e.g 6 + ☐ = 11 and 6 + 5 = 5 + ☐ 6 + 5 = ☐ + 4 6 + 5 = 11	https://vimeo.com/194169143

Pitch and expectation in Year 1

Children begin to add units together using physical objects e.g. Maths makes sense cups, counters, Numicon shapes. They count each object to find how many altogether. Teacher models the language e.g. '3 cups add 6 cups equals 9 cups altogether'. They begin to record by drawing pictures/marks. The teacher models what the adding of two groups looks like in a number sentence. The children begin to copy these number sentences onto whiteboards whilst still using objects to add.

The children become more independent and start to write number sentences into their maths books (squared maths paper) ensuring one digit in each box. Note: Leave a line after each number sentence for children to polish if needed. Children begin to add numbers that bridge 10 using the same strategies.

Introduce language of tens and units. Continue to use objects e.g. Numicon. Children start to add a 1 digit number to a 2 digit number within 20. Objects are still used to help the addition process. Begin to bridge 20.

Children are shown how to add using a number line. They record their findings orally to begin with before moving on to drawing the jumps themselves. Note: Each jump is one unit. Partial numberlines are then used as a transition to open number lines.

Challenge in Year 1: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth.

Year 2: Addition

Focus: Adding two 2 digit numbers

In year 2 children will move onto using an open number line to add 1 and 2 digit numbers. They will learn how to partition 2 digit numbers.

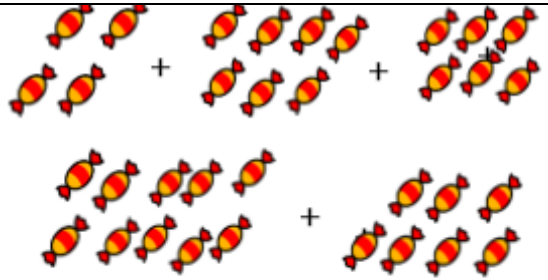
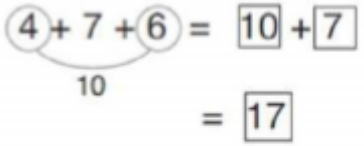
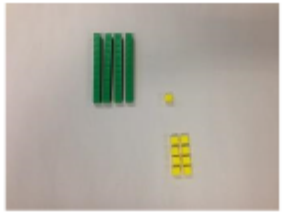
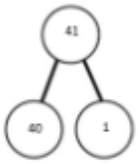
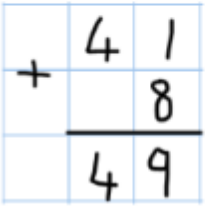
Key vocabulary

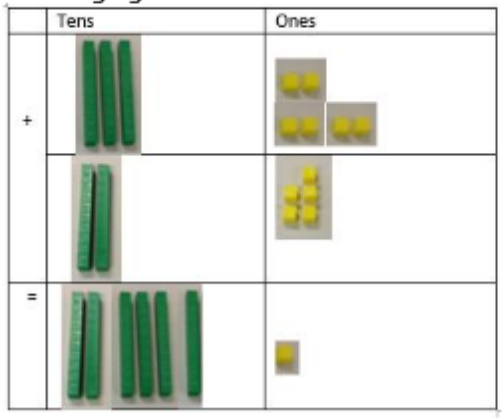
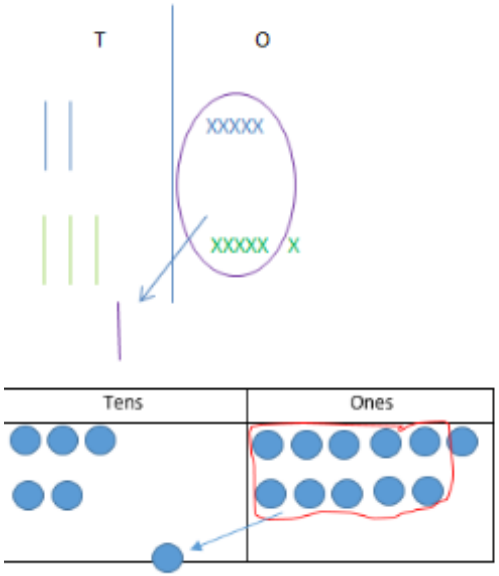
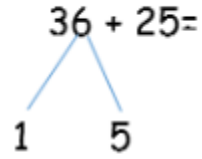
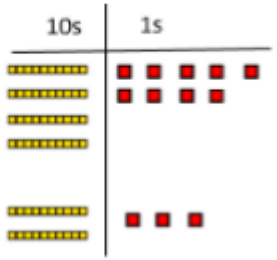
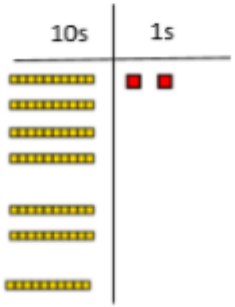
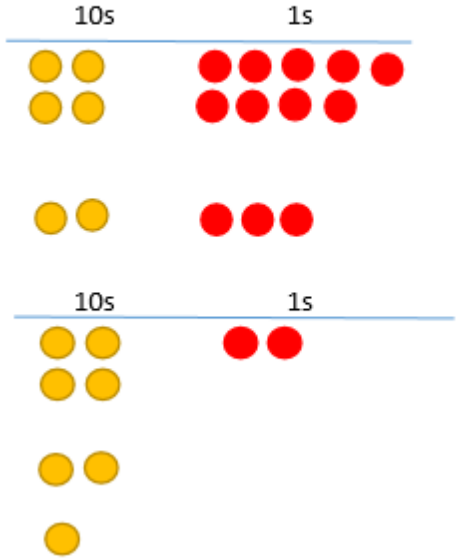
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

Key Skills for addition at Year 2

Mental Calculation	Written Calculation
? Locate any 2-digit number on a landmarked line and use this to compare numbers. ? Identify any number on the 1-100 number grid; understand that each number is a multiple of ten and some ones, e.g. 54 is 50 and 4 more. ? Know securely number pairs for all the numbers up to and including 12 ? Count in steps of 2, 5, and 10 from 0. ? Know different unit patterns when not crossing a ten, e.g. 4 + 3 = 7, 14 + 3 = 17, 24 + 3 = 27 ? Begin to recognise unit patterns when crossing a ten, e.g. 5 + 6 = 11 ? Know pairs with a total of 20 and multiples of 10 to 100 ? Count on in ones and tens from any given 2-digit number ? Know that adding can be done in any order	? Record comparisons < and >, e.g. 56 > 39. ? Add two single digit numbers (8 + 7) by counting up; add two 2-digit numbers which total less than 100 by counting on in tens and ones, e.g. 54 + 37 as 54 + 30 + 7. ? Add two or three single-digit numbers ? Add a single-digit number to any 2-digit number using number facts, including bridging multiples of 10. Add 10 and small multiples of 10 to any given 2-digit number ? Add any pair of 2-digit numbers ? Solve problems with addition using concrete objects, pictorial representations, involving numbers, quantities and measures, applying written and mental methods

Area of addition	Concrete	Pictorial	Abstract	Video link

Adding three single digit numbers	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.	https://vimeo.com/194178049
Column method without regrouping	**TO + O using base 10**. Continue to develop understanding of partitioning and place value 41 + 8 	Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 	https://vimeo.com/194170892

	TO + TO using base 10. Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging. 36 + 25 	This could be done one of two ways: 	Looking for ways to make 10  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: 36 +25 61	
Column method with regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for 1 ten. 	Using place value counters, children can draw the counters to help them to solve additions. 	40 + 9 20 + 3 60 + 12 = 72	https://vimeo.com/194173017

Pitch and expectation in Year 2

The children will move onto adding using an open numberline to add a 1 digit number to a rounded 2 digit number e.g. 20 + 7 = . When using this method securely, children move on to bridge 10 whilst still adding units e.g. 27 + 5 =

Once the children can confidently add a 1 digit number to a 2 digit number they can move on to adding two 2 digit numbers. To make it simpler for them they should start by adding rounded tens e.g. 21 + 10

Extend by adding multiple tens e.g. 21 + 20 Children will build on their number knowledge by partitioning 2 digit numbers into tens and units so they are ready for the next step. The children must be confident in their understanding of place value before moving on. Adding two 2 digit numbers using an open number line using their prior knowledge of adding tens and units. The children only need to partition the smaller number to add. 48 + 16 = 64 (bridging the 10) When the children have secured this skill they can begin to add numbers that bridge through the next ten (use 100 squares etc to help with number knowledge).

Challenge in Year 2: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth.

Year 3: Addition

Focus: Adding with numbers up to 3 digits

In year 3 we will move to the traditional column method and to support this, children will first apply their partitioning skills to the partitioning column method

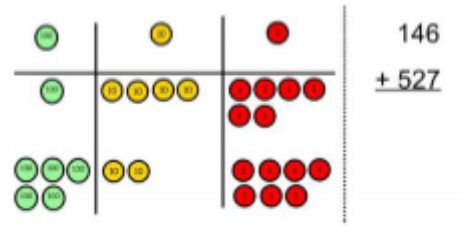
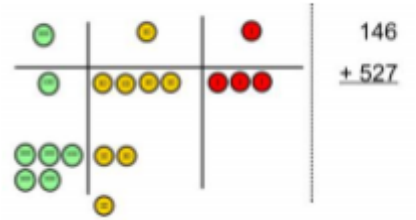
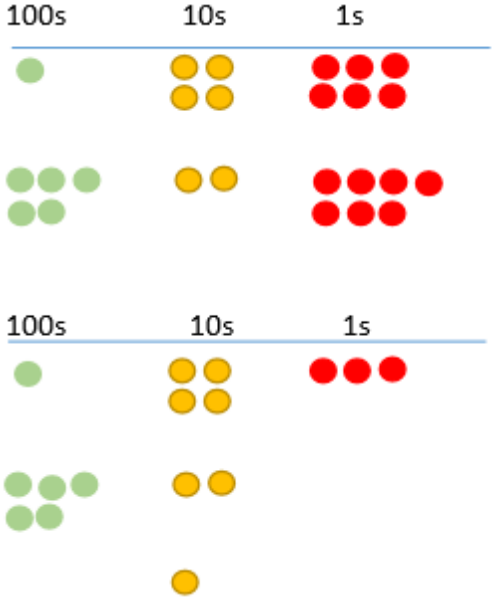
Key vocabulary

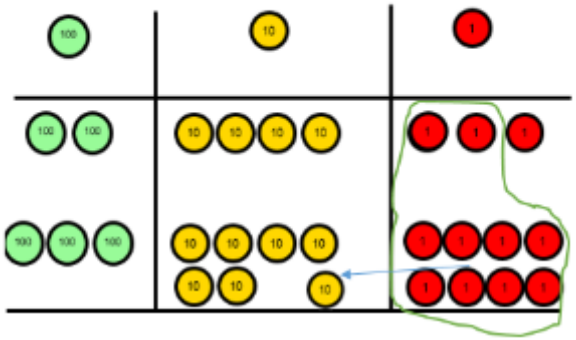
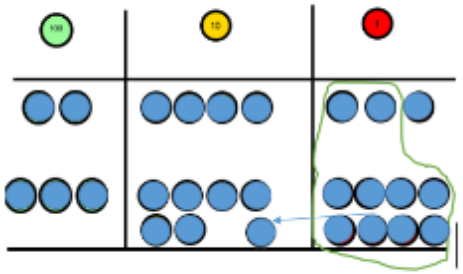
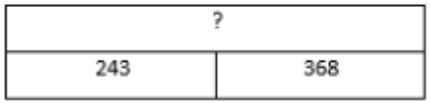
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact

Key Skills for addition at Year 3

Mental Calculation	Written Calculation
Know pairs with each total to 20 Know pairs of multiples of 10 with a total of 100 Add any two 2-digit numbers by counting on in 10s and 1s or by using partitioning Add multiples and near multiples of 10 and 100 Add 1,10, 100 to 3-digit numbers Understand place value in 3-digit numbers Perform place value additions without a struggle. (E.g. 300 + 8 + 50 = 358) Add pairs of 'friendly' 3-digit numbers mentally, e.g. 320 + 450 Use place value and number facts to add a 1-digit or 2-digit number to a 3-digit number number. (E.g. 104 + 56 is 160 since 104+50=154 and 6+4=10 and	Begin to add amounts of money using partitioning. Solve problems with addition using number facts, place value, missing numbers. *Use column method with regrouping; using understanding of place value in three digit numbers.*

676 + 8 is 684 since 8=4+4 and 76+4+4=84)	

Area of addition	Concrete	Pictorial	Abstract	Video link
Column method with regrouping	Use base ten and other representations, such as counters, to embed understanding of regrouping Make both numbers on a place value grid.  Add up the units and exchange 10 ones for 1 ten.  As children move on to decimals, money and decimal place value counters can be used to support learning.	 Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.	100 + 40 + 6 500 + 20 + 7 600 + 70 + 3 = 673 As the children progress, they will move from the expanded to the compacted method. 146 + 527 673	https://vimeo.com/194173017

	Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract 	Chidren to represent the counters e.g. like the image below  If the children are completing a word problem, draw a bar model to represent what it's asking them to do 	243 +368 611 1 1	

Pitch and expectation in Year 3

Introduce the partitioning column method with numbers that do not bridge so children become confident with the method itself.
Please Note: Start by adding the units first.
337 + 188 = 525 300 + 30 + 7 100 + 80 + 8 400 + 110 + 15 = 525
Once confident, children can start using the partitioning column method to solve problems that bridge the tens and hundreds boundaries.
343 + 116 400 50 + 9 459
Children can use the expanded column method for addition.
116 + 343= 459 343 + 116 459
Now children are ready to move on to the traditional column methods. Introduce this initially with numbers that do not bridge any boundaries. It is important children remember that it is three hundred add one hundred, NOT 3 + 1!
245 + 84 = 329 245 + 84 329 1
Once the method is secure children are now ready to be introduced to 'carrying' which happens when bridging in the column method. Make sure children add the units first and 'carry' numbers under the bottom line.

Challenge in Year 3: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth.

Year 4: Addition

Focus: Adding with numbers up to 4 digits

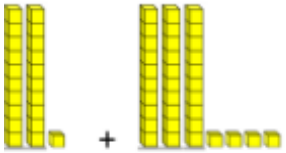
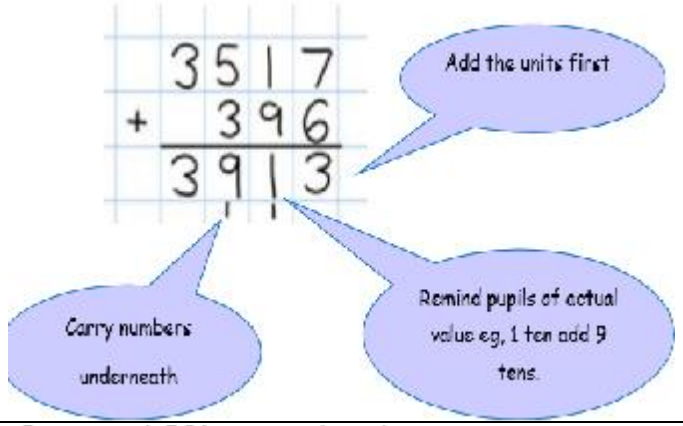
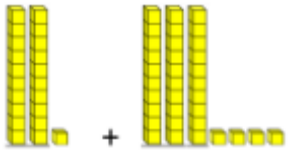
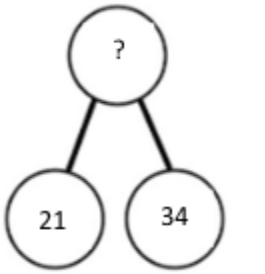
In year 4 children will consolidate their use of the traditional column method and will be able to use it confidently to add numbers up to 4 digits. This could include carrying units, tens and hundreds.

Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse

Key Skills for addition at Year 4

Mental Calculation	Written Calculation
• Select appropriate method, mental, jottings, written—and explain why • Perform inverse operations to check	
• Add any two 2-digit numbers by partitioning or counting on • Know by heart/quickly derive number bonds to 100 (eg 32 + 68) and to £1 (64p + 36p) • Add to the next hundred, pound and whole number. (E.g. 234 + 66 = 300, 3.4 + 0.6 = 4) • Perform place value additions without a struggle. (E.g. 300 + 8 + 50 + 4000 = 4358) • Add multiples and near multiples of 10, 100 and 1000. • Add £1, 10p, 1p to amounts of money • Use place value and number facts to add 1-, 2-, 3-and 4-digit numbers where a mental calculation is appropriate'. (E.g. 4004 + 156 by knowing that 6+4=10 and that 4004+150= 4154 so total is 4160) • Continue to practise a wide range of mental addition strategies eg. Round and adjust, near doubles, numbers bonds, partitioning and recombining	• Solve 2-step problems in context • *Use column method with regrouping to accurately up to pairs of four digit numbers.*

Area of addition	Concrete	Pictorial	Abstract	Video link
Continue to use compact column method adding units first and carrying underneath the calculation.	Base ten to show process of carrying/ exchanging 	Squares, sticks and dots for represent base ten		https://vimeo.com/194173804
Improve fluency and variation through reasoning and problem solving with simple addition	 **Always use missing digit problems too:** 		Sam saved £21 one week and £34 another. How much did he save in total? 21+34=55. Prove it! (reasoning but the children need to be fluent in representing this) 21 +34 ____ **21 + 34 =** ☐ = 21 + 34 What's the sum of twenty one and thirty four?	

Pitch and expectation in Year 4

Children should already be familiar with the column method from year 3 but it is very important to go over the method again ensuring children understand why

they start with the units, have to carry a number etc.
Please Note: 1) The units must be added first! 2) 'Carry' numbers underneath the bottom line! 3) Reinforce the place value! It is not 6 add 8, it is 6 tens add 8 tens!

Challenge in Year 4: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth.

Year 5: Addition

Focus: Adding with more than 4 digits

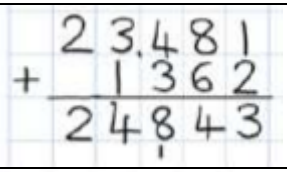
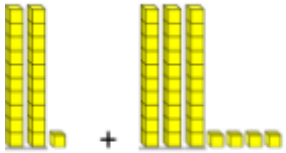
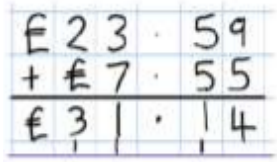
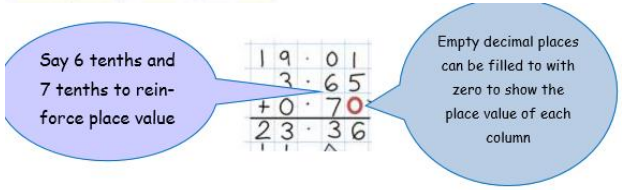
In year 5 children will now use the column method to add decimal numbers in the context of money and measures. It is important that children have place value skills beyond 4 digits here and fully understand what a decimal number represents.

Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths.

Key Skills for addition at Year 5

Mental Calculation	Written Calculation
Locate 5 and 6 digit numbers on a landmarked line; use this to compare/order numbers. Round to ten, a hundred, a thousand or ten thousand. Use rounding to check accuracy Add or subtract 0.1 or 0.01 to/from any decimal number with confidence, e.g. 5.83 + 0.01 or 4.83 – 0.1 Add and subtract mentally with confidence – where the numbers are less than 100 or the calculation relies upon simple addition and place value. Understand a one-place decimal number as a number of tenths and a two-place decimal number as a number of hundredths.	Confidently add numbers with more than 4-digits using a secure written method, including adding 'piles' of numbers Use inverse to check calculations

Area of addition	Concrete	Pictorial	Abstract	Video link
Add numbers with more than four digits	Base ten/ place value digits	Base ten/ place value digits		https://vimeo.com/194173804
Use column addition to add any pair of two-place decimal numbers including amounts of money and measure	 Base ten can be used to represent whole numbers (square) tenths (sticks) and hundredths (dots) Pounds, ten pence and pennies are also very useful	Base ten: squares, sticks and dots Pound coins, ten pence, pennies can help to show the 'missing zeros'	 	

Pitch and expectation in Year 5

The decimal point needs to be lined up just like all of the other place value columns and must be remembered in the answer column. It is important children understand why this is and get into this habit very quickly.
Children should be working with numbers greater than 4 digits including numbers in the ten thousands and hundred thousands.
Children need to start using the column method to add more than two values, still considering place value very carefully.
Please Note: 1) It is important that children say 6 tenths add 7 tenths so they understand that they are adding part of a number not a whole number. 2) Empty places should be filled with a zero to show the value of that place

Challenge in Year 5: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth.

Year 6: Addition

Focus: Adding several numbers with an increasing level of complexity

In year 6 children need to use all the previous adding skills developed to add several numbers with a variety of different decimal places. Many of these problems will be in the context of money or measures

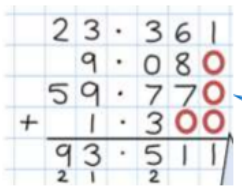
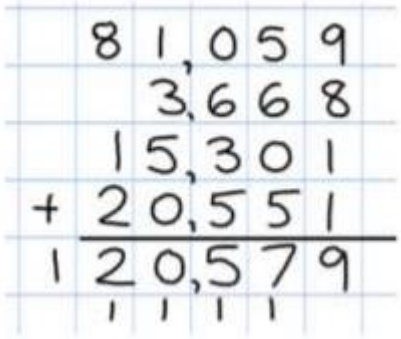
Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths.

Key Skills for addition at Year 6

Mental Calculation	Written Calculation
Add mentally with confidence using larger numbers and calculations of increasing complexity Perform mental calculations, including with mixed operations and large numbers, using a range of strategies Use estimation to check the validity of an answer.	Add several large numbers using written addition Add several large or decimal numbers using written addition Solve multi-step problems Use inverse to check the validity of an answer

Area of addition	Concrete	Pictorial	Abstract	Video link

Add several numbers of increasing complexity including money, measure, and decimal with different numbers of decimal places	Continue to pick p on misconceptions with base ten and coins. Illustrations of place value in context	Pound coins, ten pence, pennies can help to show the 'missing zeros'	 Tenths, hundredths and thousandths should be correctly aligned, with the decimal point aligned vertically, including in the answer. Empty decimal places can be filled to with zero to show the place value of each column 	https://vimeo.com/194173804

Pitch and expectation in Year 6

Children need to use their knowledge of the decimal point to line up their amounts correctly in the column. Zeroes should be added to support place value, showing that there is no value to add.
Children should also continue to add multiple integers with 4 digits or more.

Challenge in Year 6: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth.

Reception: Subtraction

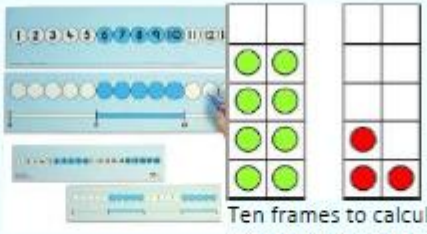
Focus:

In Reception children will be encouraged to develop a fascination with numbers; solving and creating everyday maths problems. They will use mathematical skills as part of real-life activities.

Key vocabulary

Less, less than, fewer, take-away, how many are left? count back

Reception Key skills: Mental Calculation	Reception Key skills: Written Calculation
• Finds one fewer than a group of up to five and then up to ten objects and then is able to say one fewer than a given number to 20. • Using quantities and objects they can remove a set of objects and say how many are left. • Using objects they can subtract two single-digit numbers and can count back to find the answer. • Children verbalise the calculations they are doing. • Children start to use the vocabulary of subtraction.	• There is no requirement for children to make written recording of their work but children can be encouraged to make their own jottings, drawings to explain what they are doing / have done. • Model ways to record using standard notation when appropriate.

Mental calculation	 Number tracks the objects can be place in to compare difference.  Ten frames to calculate the difference. How many have I taken away, how many are left from 10?  Bead strings for counting on an back and comparing difference A full range of different objects to count and remove objects from to find how many are left or to compare how many more if you have two groups, natural objects, seeds, sticks, pine cones, straws, counters, dice, lego bricks, people, small animals etc.. Numberlines to count backwards on such as counting how many days left until Christmas etc.  Outdoor resources Songs and rhymes that take on one away each time.(Five little ducks, five speckled frogs, five current buns, ..)

Year 1: subtraction

Focus: Subtracting with 2 digit numbers

In year 2 children will start to use blank numberlines to subtract by counting back which will greatly support the development of mental subtraction skills. Base 10 is also a super subtraction tool and should be used alongside blank numberline methods

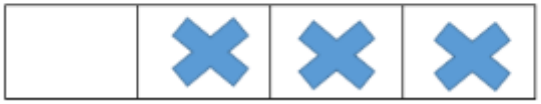
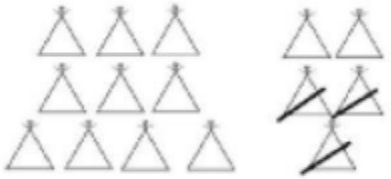
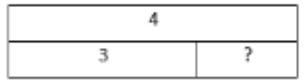
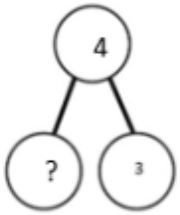
Key vocabulary

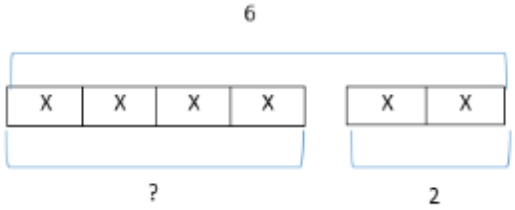
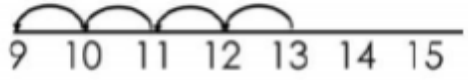
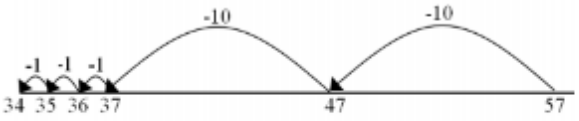
equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…

Key Skills for subtraction at Year 1

Mental calculation	Written calculation
Give a number, say one less Count back in ones to from 100 and from any single-digit or 2-digit number. Count back in tens from any 2-digit number Locate any number on a 1-100 grid or a beaded line 0-100. Know number bonds to 10, also know what is left if objects are taken from 10, e.g. 10 fingers, fold down 4, leaves 6 standing Solve one-step problems involving subtraction, using concrete objects (bead strings, objects, cubes) and pictures, and missing number problems	Recognise the - and = signs, and use these to read and write simple subtractions.

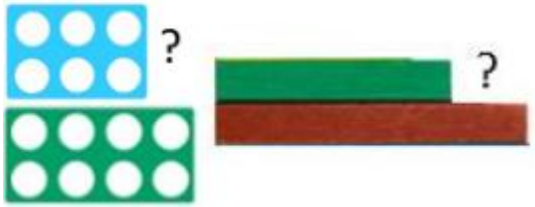
Area of subtraction	Concrete	Pictorial	Abstract	Video link

Take away ones	Physically taking away and removing objects from a whole (use various objects too) rather than crossing out - children will physically remove the objects 4 – 3 = 1  Use physical objects, counters, cubes etc. to show how objects can be taken away. 4 – 2 = 2 	Children to draw the concrete resources they are using and cross out.  Use of the bar model:  Cross out drawn objects to show what has been taken away. 4 – 2 = 2   15 – 3 = 12	4- 3 = □ = 4 - 3  	https://vimeo.com/196549268

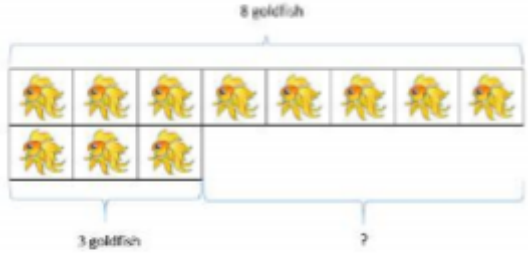
Counting back	 Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 = 9 Use counters and move them away from the group as you take them away counting backwards as you go. 	Children to represent what they see pictorially e.g.  Count back on a number line or number track  Start at the bigger number and count back the smaller number, showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.	https://vimeo.com/196550163

Find the difference

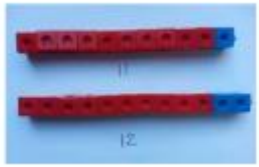
Finding the difference (using cubes, numicon or Cuisenaire rods, other objects can also be used)



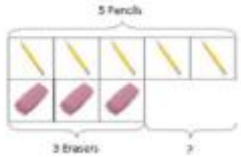
Compare amounts and objects to find the difference.



Use cubes to build towers or make bars to find the difference. Use basic bar models with items to find the difference.



Use cubes to build towers or make bars to find the difference

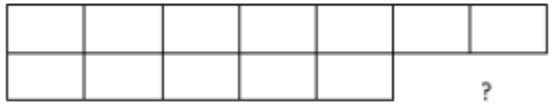


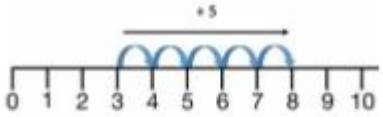
Use basic bar models with items to find the difference

Children to draw the cubes/ other concrete objects which they have used

XXXXXXXX
XXXXXX

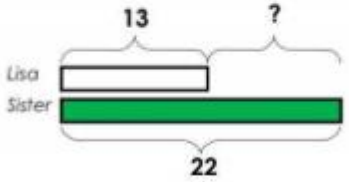
Use of the bar model





Count on to find the difference.

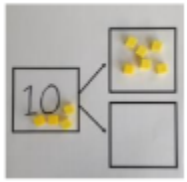
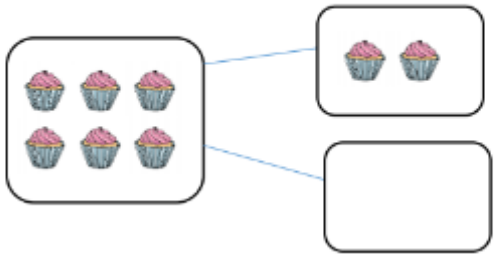
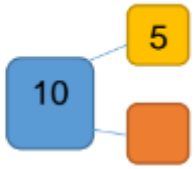
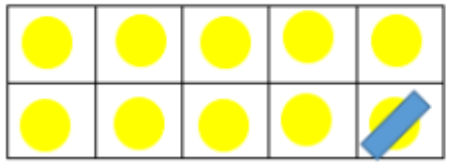
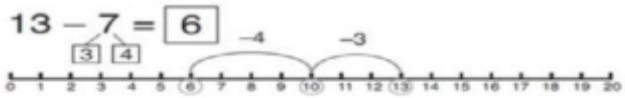
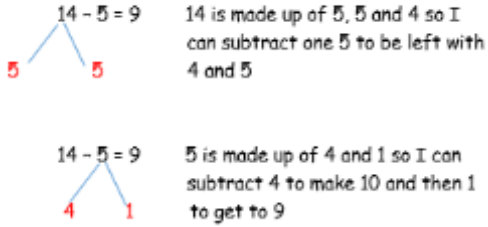
Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.



Draw bars to find the difference between 2 numbers.

Hannah has 8 goldfish.
Helen has 3 goldfish.
Find the difference between the number of goldfish the girls have.

https://vimeo.com/196552677

Part part, whole whole	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.	https://vimeo.com/196551390
Make ten	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	Children to present the ten frame pictorially    Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	14 – 5 = 9 You also want children to see related facts e.g. 15 – 9 = 5 Children to represent how they have solved it e.g.  16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?	https://vimeo.com/196553712

Pitch and expectation in Year 1

Children begin to subtract units from a large group using physical objects e.g. Maths makes sense cups, counters, Numicon shapes. They count each object to find how many left. Teacher models the language e.g. '6 cups take away 3 cups equals 3 cups'. They begin to record by drawing pictures/marks. The teacher models what the subtraction looks like in a number sentence. The children begin to copy these number sentences onto whiteboards whilst still using objects to help them subtract. The children become more independent and start to write number sentences into their maths books (squared maths paper) ensuring one digit in each box. Note: Leave a line after each number sentence for children to polish if needed. Children begin to subtract numbers that bridge 10 using the

same strategies. A 1digit number is subtracted from a 2 digit number. Introduce language of tens and units. Continue to use objects e.g. Numicon. Children are now shown how to subtract using a number line. They record their findings orally to begin with before moving on to drawing the jumps themselves. Note: Biggest number is circled and children jump back along the number line to find the answer. Jumps are one unit each. Partial numberlines are then used as a transition to open number lines.
Key

Challenge in Year 1: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 2: subtraction

Focus: Subtracting with 2 digit numbers

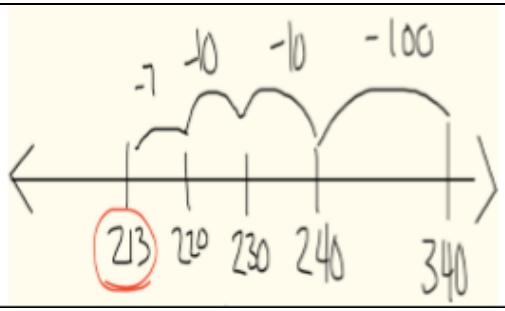
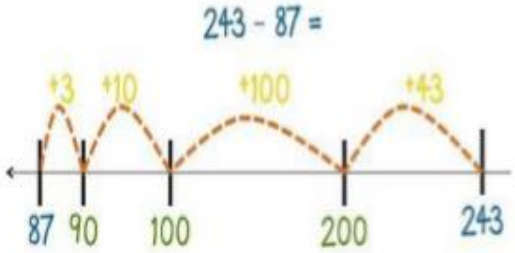
In year 2 children will start to use blank numberlines to subtract by counting back which will greatly support the development of mental subtraction skills. Base 10 is also a super subtraction tool and should be used alongside blank numberline methods.

Key vocabulary

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…, difference, count on, strategy, partition, tens units

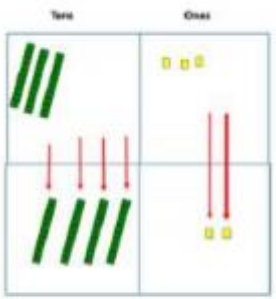
Key Skills for subtraction at Year 2

Mental Calculation	Written Calculation
Recognise that addition and subtraction are inverse operations and understand that 10 – 4 = 6 as well as 6 + 4 = 10. Count back in ones or tens to take away, e.g. 27 – 3 = or 54 – 20 =. Begin to count up to find a difference between two numbers with a small gap (42 – 38). Know when to count on and when to count back Recall and use subtraction facts to 20 fluently And derive and use related fact to 100 Subtract using concrete objects, pictorial representations, 100 squares, Dienes, Numicon and mentally, including a 2-digit number and ones, a 2-digit numbers and tens, and two 2-digit numbers Use inverse to check calculations.	*Begin using column subtraction for pairs of 2 digit numbers with some regrouping.*

Area of subtraction	Concrete	Pictorial	Abstract	Video link
Continuation of the use of a number line		 	Children will continue to subtract on a numberline using efficient jumps and now apply these to 3 digit number problems. Here is an efficient example of 340 – 127= Counting on will also be used for problems greater than 100 using efficient jumps, the use of 100 square can support children's understanding of this method.	

Column method without regrouping

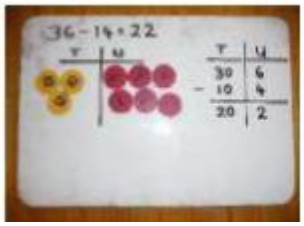
75 – 42 = 33



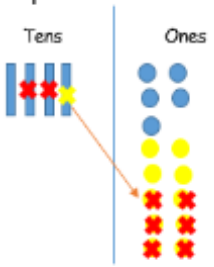
Use Base 10 to make the bigger number then take the smaller number away.

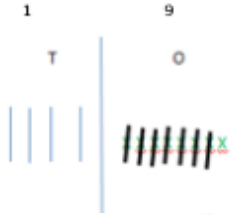
Show how you partition numbers to subtract.

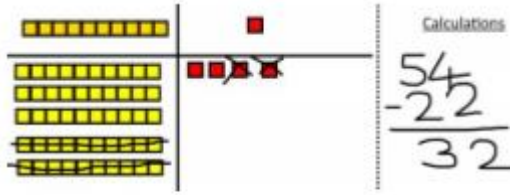
Again make the larger number first.



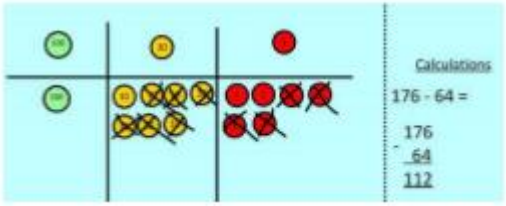
Represent the base 10 pictorially

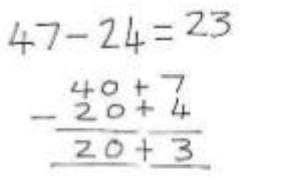




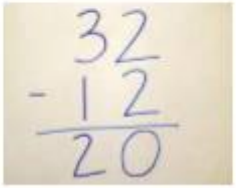


Draw the Base 10 or place value counters alongside the written calculation to help to show working.





This will lead to a clear written column subtraction.



https://vimeo.com/196556686

Pitch and expectation in Year 2

Once the children are confident using a tracked numberline, they will be shown how to use a blank numberline. They will start by subtracting units in (backward) jumps of one.

Children now bridge 10 when subtracting units from a two digit number under 20.

Partitioning is taught (or recapped) so that children can start to subtract two 2 digit numbers.
Children are now ready to subtract tens and units on an open number line. Note: Jump large tens and small units. Biggest number goes at the end of the numberline. Only the smaller number needs to be partitioned. Continue to increase difficulty by subtracting larger numbers with multiple tens. Note: Work with numbers up to 100.
Key

Challenge in Year 2: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 3: subtraction

Focus: Subtracting with 2 and 3 digit numbers

Children will consolidate their knowledge of counting back and counting on using a blank numberline to subtract. They will use these methods both written and mentally. Once children become fully confident they will be ready to move on to the partitioning column method of subtraction.

Key vocabulary

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…, difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds

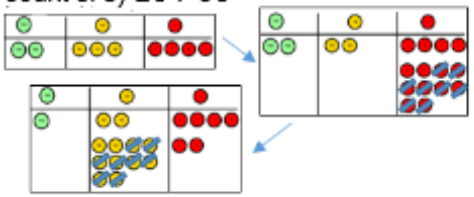
Key Skills for subtraction at Year 3

Mental Calculation	Written Calculation
Understand place value in 3-digit numbers; add and subtract 1s, 10s or 100s without difficulty; use this to add and subtract multiples of 1, 10, 100 to/from 3-digit numbers. Mentally subtract any pair of 2 digit numbers, e.g. 75 – 58 Recognise that there are two ways of completing subtractions, either by counting up (using ENL) or by counting back, e.g. 54 – 3 (counting up) Subtract mentally using place value and number bonds, eg. 347-5, 34740, 347-100)	Begin using partitioning model of column subtraction with exchanging.

Area of subtraction	Concrete	Pictorial	Abstract	Video link

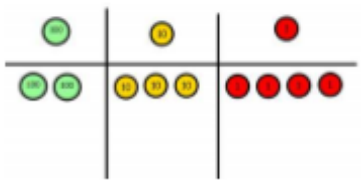
Column method with regrouping

Column method (using place value counters) 234-88



Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

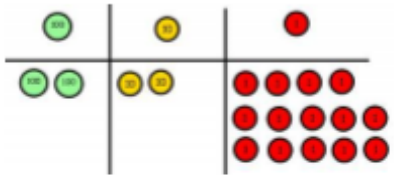
Make the larger number with the place value counters



Calculations

234
- 88

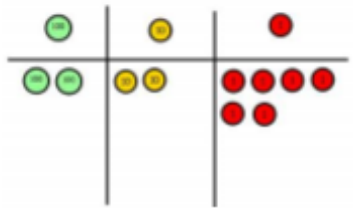
Start with the ones, can I take away 8 from 4 easily? I need to exchange 1 of my tens for 10 ones.



Calculations

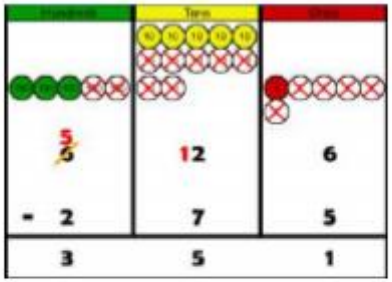
234
- 88

Now I can subtract my ones.



Calculations

234
- 88



5	12	6
- 2	7	5
3	5	1

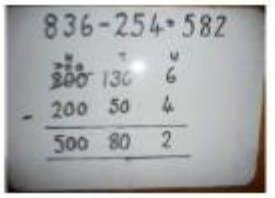
Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

It's crucial that the children understand that when they have exchanged the 10 they still have 45. 45 = 30 + 15

	3 4	5
-	2	6
	1	9



Children can start their formal written method by partitioning the number into clear place value columns.

Moving forward the children use a more compact method.

This will lead to an understanding of subtracting any number including decimals.

		5	12		1
	2	6	3	.	0
-		2	6	.	5
	2	3	6	.	5

https://vimeo.com/196558650

Pitch and expectation in Year 3

Children will continue to subtract on a numberline using efficient jumps and now apply these to 3 digit number problems. Here is an efficient example of 340 – 127=

Counting on will also be used for problems greater than 100 using efficient jumps, the use of 100 square can support children's understanding of this method.

Children will now have the mental skills required to approach the partitioning column method of subtraction. At first they should attempt this where no exchanging is required. Here is an example for 89 – 35 = 54 Through practical subtraction children should be introduced to exchanging. Base 10 is a vital tool here as is a solid grounding with partitioning in different ways. It is important children realize that the value has not changed, we have just partitioned in a different way. As you can see here for 72 – 47, before subtracting 7 units, a tens row will need to be exchanged for 10 units. Children who are secure with the concept of 'exchanging' should now be able to use the partitioning column method to subtract any 2

Challenge in Year 3: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 4: subtraction

Focus: Subtracting with numbers up to 4 digits

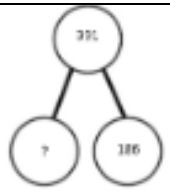
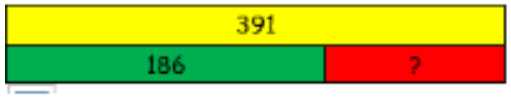
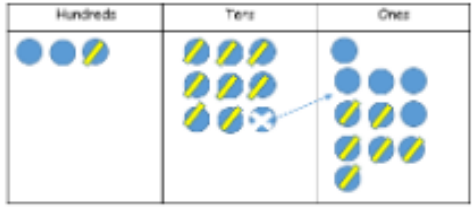
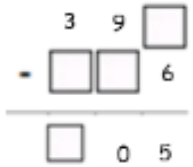
Children will consolidate their knowledge of the partitioning column method for subtraction with 4 digit numbers including those where exchanging is required. Once they are secure with this they will move on to the compact (traditional) method of column subtraction.

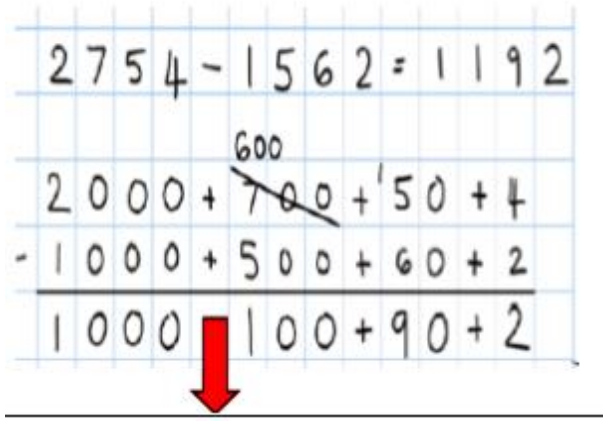
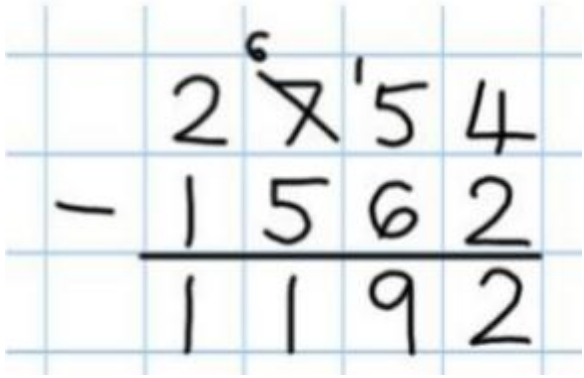
Key vocabulary

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…, difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds, inverse

Key Skills for subtraction at Year 4

Mental Calculation	Written Calculation
Mentally subtract any pair of two digit numbers. Practise mental subtraction strategies, eg. Round and adjust (37—9), using place value Use counting on in the context of money and also when subtracting from numbers ending in zeros eg 4000-372 Count backwards through zero, using negative numbers	Subtract 3 digit numbers from 3 digit numbers using counting on, e.g. 426 – 278 by jumping along a line from 278 to 426

Area of subtraction	Concrete	Pictorial	Abstract	Video link
Using fluency, reasoning and problem solving	Range of base ten, place value cards, etc.	  What's the calculation? What's the answer?   	Raj spent £391, Timmy spent £186. How much more did Raj spend? I had 391 metres to run. After 186 I stopped. How many metres do I have left to run? 391 – 186 □ = 391 – 186 391 -186 ____ Find the difference ebtween 391 and 186 Subtract 186 from 391. What is 186 less than 391?	

	Range of base ten, place value cards, etc.		Children will consolidate their learning of the partitioning column method of subtraction and exchanging by solving calculations with more complex numbers. Place value counters will come in handy here when building children's confidence. Money can also be partitioned for subtraction e.g. £1 + 30 + 5 - £1 + 10 + 2 =	
	Range of base ten, place value cards, etc.		Once confident children are ready to move on to the compact method of subtraction. Encourage children to complete a calculation in the partitioning column methods and then model compact method. See if children can see how they are linked and discuss which is simpler. Although this is seen as the 'easiest' method it does not mean that it is necessarily the best method and they need to carefully select the best method for the problem they are solving.	https://vimeo.com/196558650

Pitch and expectation in Year 4

Children will consolidate their learning of the partitioning column method of subtraction and exchanging by solving calculations with more complex numbers. Place value counters will come in handy here when building children's confidence. Money can also be partitioned for subtraction e.g. £1 + 30 + 5 - £1 + 10 + 2 =
Once confident children are ready to move on to the compact method of subtraction. Encourage children to complete a calculation in the partitioning column methods and then model compact method. See if children can see how they are linked and discuss which is simpler. Although this is seen as the 'easiest' method it does not mean that it is necessarily the best method and they need to carefully select the best method for the problem they are solving.

Challenge in Year 4: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 5: subtraction

Focus: Subtracting with numbers beyond 4 digits including decimals

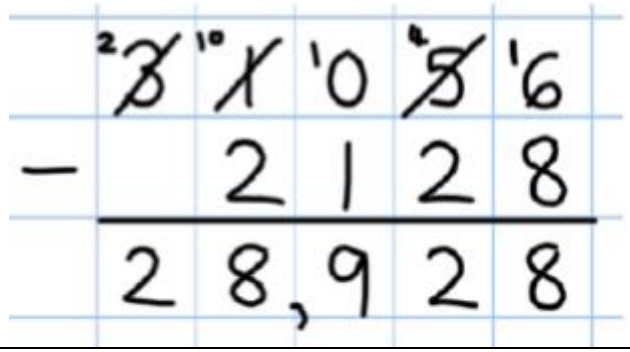
Children in year 5 will continue to use the compact column method of subtraction to solve problems including those where exchanging is required. They will subtract larger integers and begin to subtract decimal amounts.

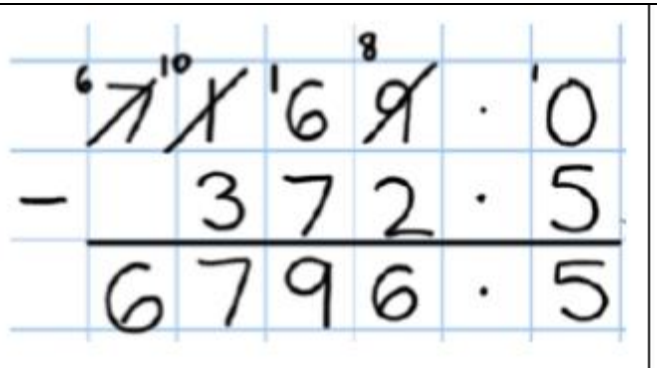
Key vocabulary

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…, difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds, inverse, tenths, hundredths, decimal point, decimal

Key Skills for subtraction at Year 5

Mental Calculation	Written Calculation
Count backwards through zero, using negative numbers Add or subtract 0.1 or 0.01 to/from any decimal number with confidence, e.g. 5.83 + 0.01 or 4.83 – 0.1 Decide which mental methods to use and explain why	Children need to utilise and consider a range of subtraction strategies, jottings and written methods before choosing how to calculate Subtract larger numbers using column subtraction or by counting up Begin to subtract decimal numbers using counting up: 6.2 – 3.5

Area of subtraction	Concrete	Pictorial	Abstract	Video link
Several exchanges	Range of base ten, place value cards, etc.		Children will come across problems where exchanging will need to take place several times to complete the problem.	https://vimeo.com/196558650

Subtraction with decimals	Range of base ten, place value cards, etc.		Once confident with large integers, children will now be ready to move onto decimal numbers including lots in the context of measures and money. Just like addition, it is important that the children line up the decimal point and understand why they are doing this. Please Note: Where there is a space in a column it is important that children add a zero so they understand the value and know what to subtract in that column.	

Pitch and expectation in Year 5

Children will come across problems where exchanging will need to take place several times to complete the problem.
Once confident with large integers, children will now be ready to move onto decimal numbers including lots in the context of measures and money. Just like addition, it is important that the children line up the decimal point and understand why they are doing this.
Please Note: Where there is a space in a column it is important that children add a zero so they understand the value and know what to subtract in that column.

Challenge in Year 5: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 6: subtraction

Focus: Subtracting with increasingly complex numbers including decimals

In year 6, children need to use mental methods and the compact column method of subtraction to solve an increasingly complex range of calculation including those with integers, those with decimals and those with mixed numbers.

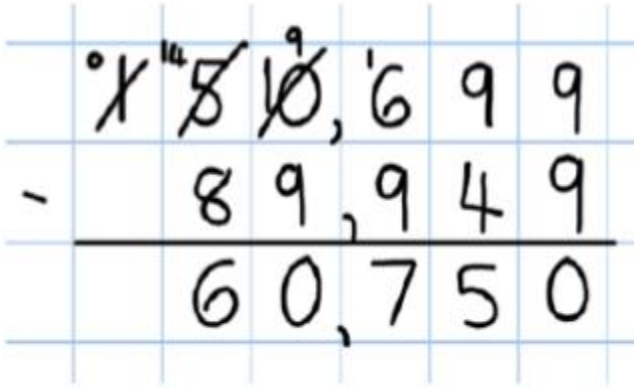
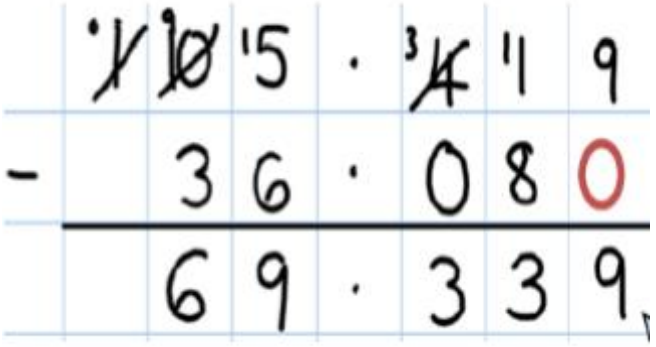
Key vocabulary

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…, difference, count on, strategy, partition, tens units, take and make, exchange, digit, value, hundreds, inverse, tenths, hundredths, decimal point,

decimal

Key Skills for subtraction at Year 6

Mental Calculation	Written Calculation
• Subtract mentally with confidence – where the numbers are less than 100 or the calculation relies upon simple subtraction and place value. Examples include: 6,723 – 400, 72 – 46, 100 – 64 • Use negative numbers in context and calculate intervals across zero • Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before deciding how to calculate • Decide which methods to use and explain why	• Subtract large numbers using column subtraction or counting up, e.g. 1323 – 758 • Subtract decimal numbers using counting up • Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before deciding how to calculate • Decide which methods to use and explain why

Area of subtraction	Concrete	Pictorial	Abstract	Video link
Several exchange			Children will use the compact method to solve problems involving integers up to 6 digits and beyond and solve problems where they will need to use 'exchanging' several times.	https://vimeo.com/196558650
Subtraction of decimals			They will also solve problems in context involving increasingly large decimals. They will need to continue using their knowledge of decimal points to line up their numbers and place zeroes in any empty places so they fully understand the value of that column.	

Pitch and expectation in Year 6

Children will use the compact method to solve problems involving integers up to 6 digits and beyond and solve problems where they will need to use 'exchanging' several times.

They will also solve problems in context involving increasingly large decimals. They will need to continue using their knowledge of decimal points to line up their numbers and place zeroes in any empty places so they fully understand the value of that column.

Challenge in Year 6: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Reception: Multiplication

Focus:

In Reception children will be encouraged to develop a fascination with numbers; solving and creating everyday maths problems. They will use mathematical skills as part of real-life activities.

Key vocabulary

Double, groups of, pairs.

Reception Key skills: Mental Calculation	Reception Key skills: Written Calculation
• Double and halve numbers up to 10 Put objects into pairs and count up in twos • Children start to use the vocabulary of doubling. • Children verbalise the calculations they are doing. • Children understand that two rows of three eggs in the box make six eggs altogether.	• There is no requirement for children to make written recording of their work but children can be encouraged to make their own jottings, drawings to explain what they are doing / have done. • Model ways to record using standard notation when appropriate.

Mental calculation	 Counting in twos 2p coins

Year 1: Multiplication

Focus: Repeated addition with objects, arrays and pictorial representations.

In year one children will be exposed to many different multiplication based activities in a variety of contexts. Much of this will be repeated addition activities or be linked to counting in 2s, 5s or 10s.

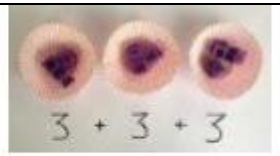
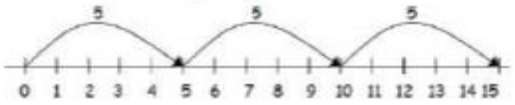
Key vocabulary

groups of, lots of, times, array, altogether, multiply, count

Key Skills for multiplication at Year 1

Mental Calculation	Written Calculation
Count in multiples of 2, 5 and 10 Recognise doubles to double 6 Solve simple one-step problems involving multiplication and division, calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Solve simple one-step problems involving multiplication and division, calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Area of multiplication	Concrete	Pictorial	Abstract	Video link
Doubling	Objects being doubled (bubble, bubble make it double)	Children's own drawings	4X2= 4+4 =	https://vimeo.com/205760888
Repeated addition	Repeated grouping/ repeated addition (does not have to be restricted to cubes) 3 x 4 or 3 lots of 4 	Children to represent the practical resources in a picture e.g. XX XX XX XX XX XX Use of a bar model for a more structured method 	3 x 4 4 +4 +4	https://vimeo.com/205761867

	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 + 2 + 2 = 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 = 6	https://vimeo.com/205761484

Pitch and expectation in Year 1

The teacher gives verbal instructions showing children how to 'multiply' the same amount of objects e.g. I give out 3 sweets and I do the same thing 4 times'. The children record pictorially. The written multiplication sentence will be modelled by the teacher and the children will start to copy onto whiteboards/into their books. 3 x 2 = Children record each number sentence by drawing the array e.g. put '3 cups on the maths table, do it two times'

3 x 2 = 6

Once children have shown a basic understanding of multiplication they will start to record in numbers, not pictorially anymore. They write each number sentence onto squared paper (maths books). Note: Objects to aid working out are available at all times until children begin to use mental recall strategies.

Challenge in Year 1: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 2: Multiplication

Focus: Multiplying using arrays and repeated addition- 2,3,4,5,10x table facts In year 2 children will be aware of simple arrays and pictorial representations and understand what they mean.

In year 2 children will develop the knowledge of how to make their own arrays to solve a problem and also how repeated addition on a numberline can get them to a solution.

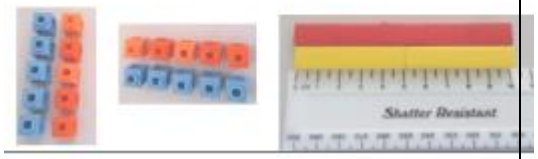
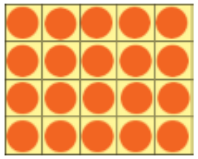
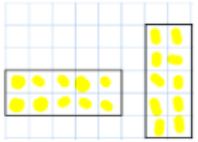
Key vocabulary

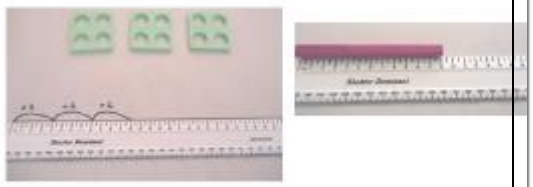
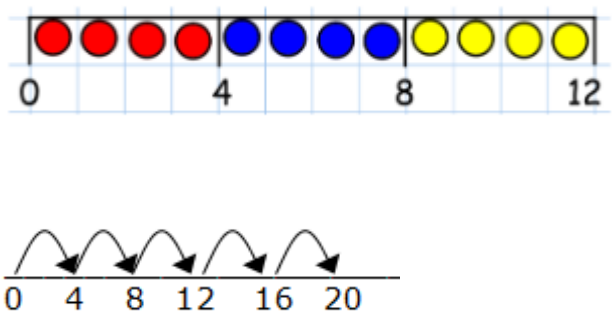
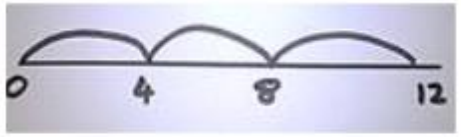
groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three

times…

Key Skills for multiplication at Year 2

Mental Calculation	Written Calculation
• Count in steps of 2, 3 and 5 from zero and in 10s from any number • Know the 2X, 5X and 10X tables and begin to say how many 10s are in 40 or how many 5s are in 30; recognise odd and even answers • Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, Numicon, mental methods and multiplication facts	• Write and calculate number statements using x and = signs • Show that multiplication can be done in any order

Area of multiplication	Concrete	Pictorial	Abstract	Video link
Arrays showing commutative multiplication	Use arrays to illustrate commutativity (counters and other objects can also be used) 2 x 5 = 5 x 2  Create arrays using counters/cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  4 × 2 = 8 2 × 4 = 8  2 × 4 = 8 4 × 2 = 8 Link arrays to area of rectangles.  Children to draw the arrays 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15 Children to be able to use an array to write a range of calculations e.g. 2 x 5 = 10 5 x 2 = 10 2 + 2 + 2 + 2 + 2 = 10 5 + 5 =10	https://vimeo.com/205762225

Repeated addition	Use number lines to show repeated groups- 3 x 4 	Represent this pictorially alongside a number line e.g: 	Abstract number line 3 x 4 = 12 	https://vimeo.com/205761867

Pitch and expectation in Year 2

Children will be shown that multiplication of two numbers can be done in any order (commutative) and will use arrays to represent this. Physical objects/drawings used to aid working out.

2+2+2+2=8 2 x 4 = 8

Repeated addition will be taught as another strategy to multiplication. Physical objects/drawings used to aid working out. Repeated addition moves on to using an open number line. Group size x number of groups = product.

e.g. 9 x 4 = 7 x 6 = (draw array/use open number line) 8 x 9 =

Children continue to master these strategies until they can confidently multiply a 1 digit number with a 1 digit number.

Challenge in Year 2: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 3: Multiplication

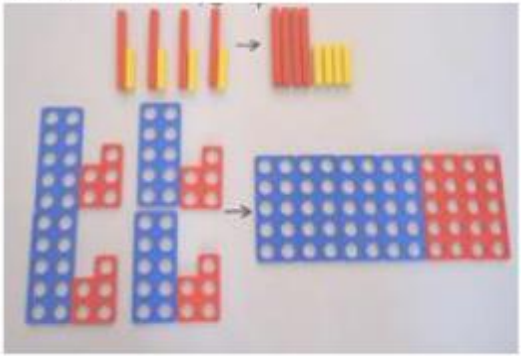
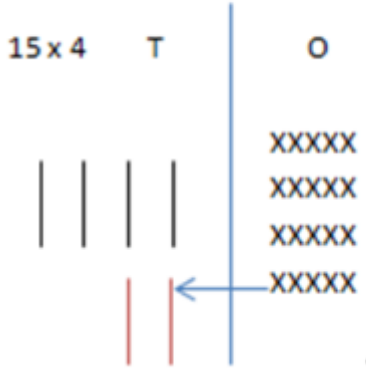
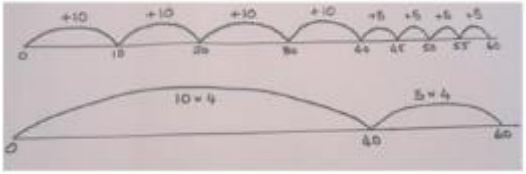
Focus: Multiplying 2 digit numbers by 1 digit numbers

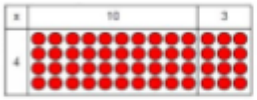
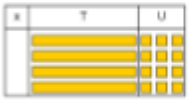
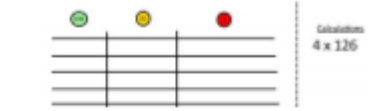
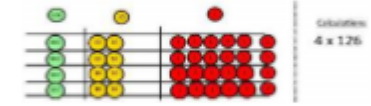
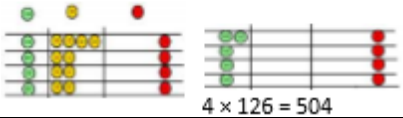
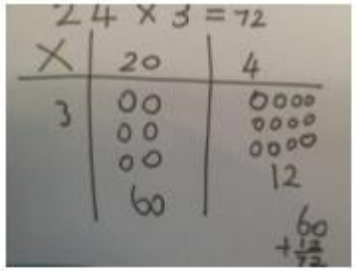
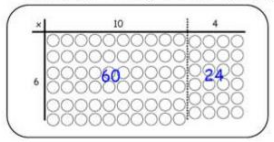
In year 3 children will move on from arrays and start using the grid method of multiplication. It is essential that before children move onto the grid method they are completely confident with all previous methods and have a solid grounding with mental methods and partitioning.

Key vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value

Key Skills for multiplication at Year 3	
Mental Calculation	**Written Calculation**
▯ Understand that multiplication is commutative, e.g. 4 x 8 is the same as 8 x 4. ▯ Know the 2x, 3x, 5x and 10x times tables. All tables need to be learned to 12th multiple. ▯ Multiply any 2-digit number by 10 or a single-digit number by 100; ▯ Understand the effect of multiplying whole numbers by 10 and 100.	▯ Multiply a 1 digit number by a 2 digit number starting to use the grid ▯ Solve multiplication problems involving missing numbers

Area of multiplication	Concrete	Pictorial	Abstract	Video link
Partitioning numbers to multiply	Partition to multiply (use numicon, base 10, Cuisenaire rods) 4 x 15 	Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like: 	Children to be encouraged to show the steps they have taken 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 	

	Formal column method with place value counters or base 10 (at the first stage- no exchanging) 3 x 23 Make 23, 3 times. See how many ones, then how many tens 	Children to represent the counters in a pictorial way 	Children to record what it is they are doing to show understanding 3 x 23 20 3 3 x 20 = 60 3 x 3 = 9 60 + 9 = 69 23 × 3 69	
Grid method	Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.  Fill each row with 126.  Add up each column, starting with the ones making any exchanges needed.  4 × 126 = 504	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below. 	Eg. 23 x 8 = 184 X | 20 | 3 8 | 160 | 24 160 + 24 = 184 Link the layout of the grid to an array initially: 	https://vimeo.com/205763255

Pitch and expectation in Year 3

The grid method should be introduced using an arrays model such as the one to the left for 14 x 6. Children need to use their partitioning skills to partition the two digit number and then use their existing knowledge of arrays to come to an answer with minimal support.

Multiplication grid method requires good organization but also a solid understanding of partitioning and multiplication facts, as you can see in the example to the left for 35 x 7. The children need to remember that once they have multiplied the partitioned parts of the number, they then need to add the two

Key

Challenge in Year 3: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 4: Multiplication

Focus: Multiplying 2 and 3 digit numbers by 1 digit numbers

In year 4 children need to use the grid method confidently to solve problems where a 2 or 3 digit number is multiplied by a one digit number. They need to move on to the use of short multiplication to solve 3 digit number multiplied by 1 digit problems.

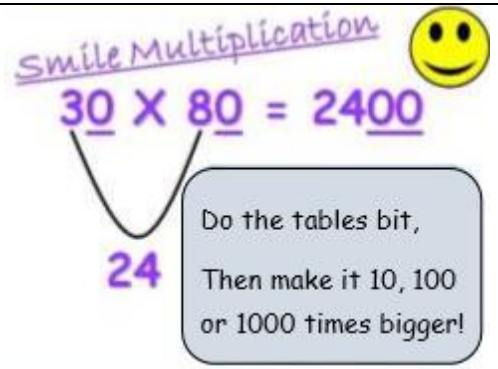
Key vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value, inverse

Key Skills for multiplication at Year 4

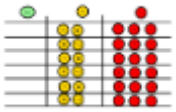
Mental Calculation	Written Calculation
• Multiply 1 and 2 digit numbers by 10, 100 and 1000; to understand place value in decimal numbers with one place. • Know and recite 2x, 3x, 4x, 5x, 9x, 10x times tables up to 12th multiple; include multiplying by 0 (e.g. 5 x 0 = 0, 7 x 0 = 0) or by 1 (e.g. 5 x 1 = 5, ½ x 1 = ½). • Use doubling as strategy for multiplying by 2, 4, 8 • Count in multiples of 6, 7, 9, 25 and 1000	• Multiply 1- digit numbers by 2-digit or friendly 3-digit numbers using grid method. • Find doubles to double 100 and beyond, using partitioning • Begin to double amounts of money

Area of multiplication	Concrete	Pictorial	Abstract	Video link

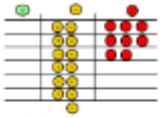
Grid method	Continue to use base ten to show the value. Understanding of value of numbers in real life context.		Developing the grid method: Eg. 136 x 5 = 680 X | 100 | 30 | 6 5 | 500 | 150 | 30 500 150 + 30 680 Encourage mental addition or use of column addition to add accurately. Start with multiplying by one digit numbers and showing the clear addition alongside the grid. X | 30 | 5 7 | 210 | 35 210 + 35 = 245 Moving forward, multiply by a 2 digit number showing the different rows within the grid method. 10 | 8 10 | 100 | 80 3 | 30 | 24 X | 1000 | 300 | 40 | 2 10 | 10000 | 3000 | 400 | 20 8 | 8000 | 2400 | 320 | 16	https://vimeo.com/205763255

Formal column method (2 digit by 1 digit)

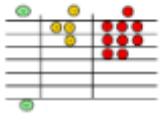
6 x 23



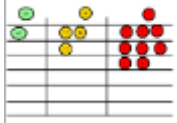
Step 1: get 6 lots of 23



Step 2: 6 x 3 is 18. Can I make an exchange? Yes! Ten ones for one ten...



Step 3: 6 x 2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred...



Step 4- what do I have in each column?

The aim is to get to the formal method but the children need to understand how it works.

$$6 \times 23 = 138$$

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

x	600	10	3
5	3000	50	15

Add up 3000, 50 and 15 to make 3065.

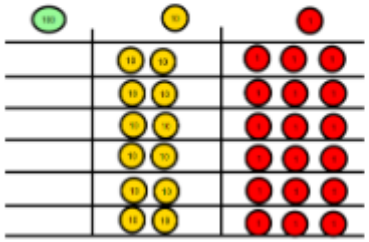
613 x 5 = 3065

The grid method is extended in year 4 so children will now multiply 3 digit numbers by 1 digit numbers. When adding the 3 answers up to create a total, column addition could be used to ensure accuracy, especially where bridging will be needed.

$$\begin{array}{r} 463 \\ \times\ \ 8 \\ \hline 3704 \\ \hline 5\ 2 \end{array}$$

The compact 'short multiplication' method is tricky and needs to be approached carefully. At first children should solve a problem using grid method and then observe the teacher solve a problem using short multiplication and make comparisons. How are they similar? Children need to go through it very slowly and carefully, unpicking each step until they are fully confident.

https://vimeo.com/205767370

To improve fluency, reasoning and problem solving.		What's the calculation? What's the answer?   With the counters, prove that 6 x 23 = 138 Why is 6 x 23 = 32 x 6?	Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? Tom saved 23p three days a week. How much did he save in 2 weeks? Find the product of 6 and 23 6 x 23 = ☐ = 6 x 23 6 × 23 = ___ 23 × 6 = ___	

Pitch and expectation in Year 4

The grid method is extended in year 4 so children will now multiply 3 digit numbers by 1 digit numbers. When adding the 3 answers up to create a total, column addition could be used to ensure accuracy, especially where bridging will be needed.

The compact 'short multiplication' method is tricky and needs to be approached carefully. At first children should solve a problem using grid method and then observe the teacher solve a problem using short multiplication and make comparisons. How are they similar? Children need to go through it very slowly and carefully, unpicking each step until they are fully confident.

Challenge in Year 4: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 5: Multiplication

Focus: Multiplying up to 4 digits by 1 or 2 digits

In year 5 children will continue to use short multiplication to solve increasingly richer problems that involve multiplying by 1 digit. They will then move on to long

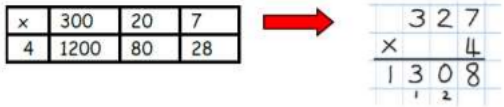
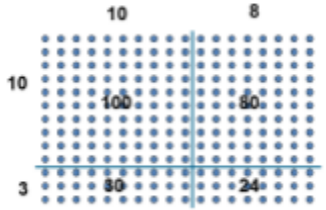
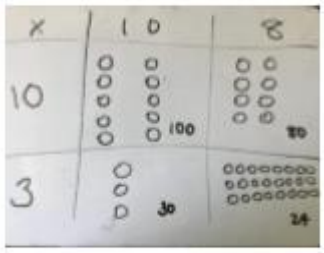
multiplication for problems that involve multiplying by 2 digits. Approximation will play an important part- with children making approximations before using long multiplication to help check their answer is correct.

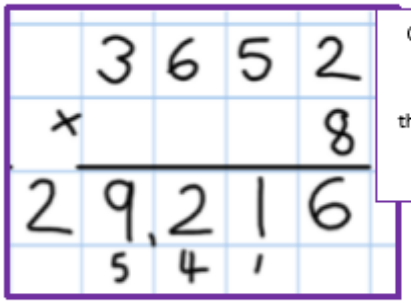
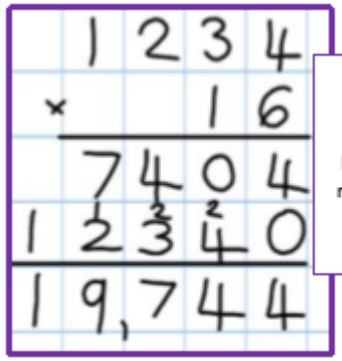
Key vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/long multiplication, 'carry'

Key Skills for multiplication at Year 5

Mental Calculation	Written Calculation
Know and recite all times tables including division facts. Identify multiples and factors, using knowledge of multiplication tables up to 12 x 12 Scale up or down by a factor of 2, 5 or 10 Multiply integers and decimals by 10, 100, 1000	Multiply 2- and 3-digit numbers by numbers ≤12 using grid method; multiply 2-digit by 2-digit numbers using grid method. Recognise and use squared, cubes and their notations

Area of multiplication	Concrete	Pictorial	Abstract	Video link
Grid method & compact method for 3 digit X 1 digit	Continue to use base ten to show the value. Understanding of value of numbers in real life context.	Base ten: squares, sticks and dots	Introduce column multiplication by comparing a grid method calculation, in order to see how the steps are related. Notice how there are less steps involved. 	https://vimeo.com/205767370
Expanded multiplication	Show the link with arrays to first introduce the expanded method. 			

			 Children will use short multiplication in a range of increasingly challenging problems. Solving using the grid method and then comparing to the short multiplication method will help cement the children's understanding of the shirt multiplication method.  Once long multiplication methods are secure, children are ready to move on to more challenging problems which require greater levels of mental calculation. The problem to the right show 1234 x 6 on the top line, 1234 x 10 on the bottom line and the total of both calculations on the final row.	https://vimeo.com/205767370

Pitch and expectation in Year 5

Children will use short multiplication in a range of increasingly challenging problems. Solving using the grid method and then comparing to the short multiplication method will help cement the children's understanding of the shirt multiplication method.

When multiplying by more than 1 digit, children need to use long multiplication. Like with short multiplication, they will solve the problem using the grid method first and then make comparisons until their understanding is secure. In the example below the top row shows 18 x 3 and the bottom shows 18 x 10. The final row shows the total of both calculations.

Once long multiplication methods are secure, children are ready to move on to more challenging problems which require greater levels of mental calculation. The problem to the right show 1234 x 6 on the top line, 1234 x 10 on the bottom line and the total of both calculations on the final row.

Challenge in Year 5: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 6: Multiplication

Focus: Consolidating short and long multiplication, multiplying decimals by 1 digit

In year 6 children will consolidate all they know about short and long multiplication before they go to Secondary school. They will also learn the new skill of using short multiplication to multiply decimal numbers to 2 decimal places. They must use rounding and place value to make approximations before calculating and use these to check validity of answers.

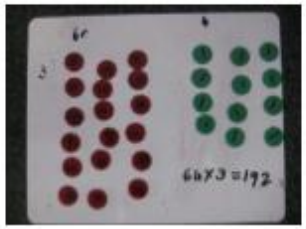
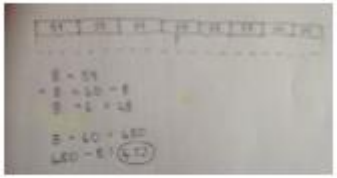
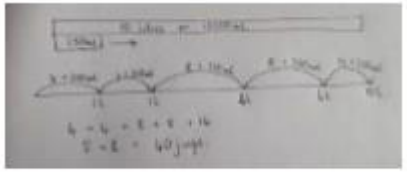
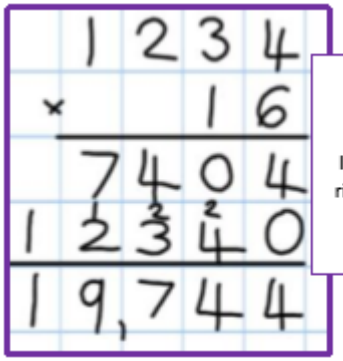
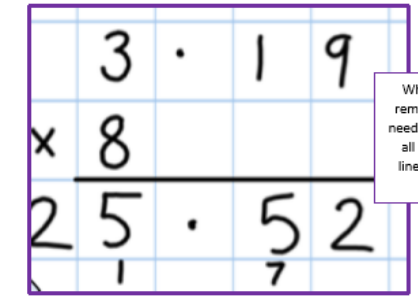
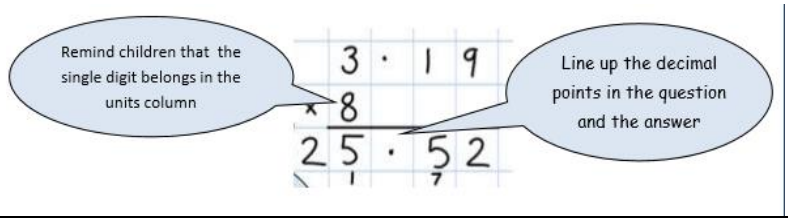
Key vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, sets of, equal groups, times as big as, once, twice, three times…, partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/long multiplication, 'carry', tenths, hundredths, decimal

Key Skills for multiplication at Year 6

Mental Calculation	Written Calculation
Recall multiplication facts up to 12 x 12 Estimate answers using rounding and approximation	Use short multiplication to multiply a 1-digit number by a number with up to 4 digits Use long multiplication to multiply a 2-digit by a number with up to 4 digits Use short multiplication to multiply a 1-digit number by a number with one or two decimal places, including amounts of money. Multiply fractions and mixed numbers by whole numbers. Multiply fractions by proper fractions. Use percentages for comparison and calculate simple percentages.

Area of multiplication	Concrete	Pictorial	Abstract	Video link

Long multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	 Once long multiplication methods are secure, children are ready to move on to more challenging problems which require greater levels of mental calculation. The problem to the right show 1234 x 6 on the top line, 1234 x 10 on the bottom line and the total of both calculations on the final row.	
Multiplying decimals			 When multiplying decimals it is important to remember that the digit you are multiplying by needs to be lined up with the ones digits. As with all decimal work, the decimal points must be lined up and the children need to have a clear understanding why that is.  Remind children that the single digit belongs in the units column Line up the decimal points in the question and the answer	

Pitch and expectation in Year 6

When multiplying decimals it is important to remember that the digit you are multiplying by needs to be lined up with the ones digits. As with all decimal work, the decimal points must be lined up and the children need to have a clear understanding why that is.

Challenge in Year 6: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Reception: Division

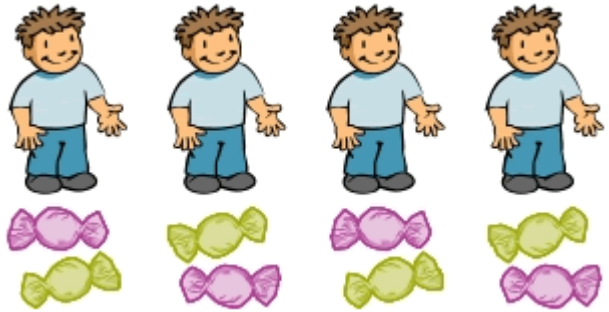
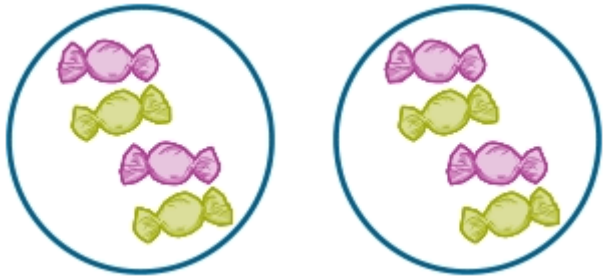
Focus:

In Reception children will be encouraged to develop a fascination with numbers; solving and creating everyday maths problems. They will use mathematical skills as part of real-life activities.

Key vocabulary

Share, how many?, groups.

Reception Key skills: Mental Calculation	Reception Key skills: Written Calculation
• Share out objects between two people and count the objects and say how many each person will get. • Children verbalise the calculations they are doing. • Children extend their thinking to 'suppose there were three people to share the bricks between instead of two. . .' • Children start to explore halving as a sharing model. Real life objects, counters etc.	• There is no requirement for children to make written recording of their work but children can be encouraged to make their own jottings, drawings to explain what they are doing / have done. • Model ways to record using standard notation when appropriate.

Mental calculation	 

Year 1: Division

Focus: Grouping and sharing small quantities without remainders As an introduction to division, children in year 1 will solve problems in familiar and relevant contexts where they have to group and share. They will use objects and pictorial representations to solve problems and they will begin to use counting in 2s, 5s and 10s to support their problems solving.

Key vocabulary

share, share equally, one each, two each…, group, groups of, lots of, array

Key Skills for division at Year 1

Mental Calculation	Written Calculation
Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities. They make connections between arrays, number patterns, and counting in twos, fives and tens.	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher

Area of division	Concrete	Pictorial	Abstract	Video link
Sharing	**I have 8 cubes, can you share them equally between two people**	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 4 ÷ 2 =  2	Share 8 buns between two people. 8 ÷ 2 = 4  	https://vimeo.com/206862682

Pitch and expectation in Year 1

The children will start by sharing objects between set groups e.g. 12 sweets shared between 3 children. They will discuss how to share equally so no group has more or less. The written division sentence will be modelled by the teacher and the children will start to copy onto whiteboards/into their books.

4 ÷ 2 = 2

Children will begin to use arrays to work out division sentences by drawing rings around each 'group'.

Challenge in Year 1: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 2: Division

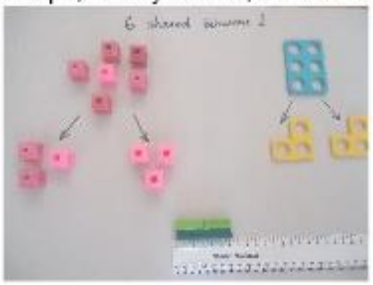
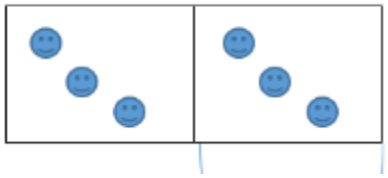
Focus: Grouping and sharing larger quantities using written methods and symbols Children will continue to use the methods of sharing and grouping in division with objects to support their understanding of arrays for sharing and grouping and the division numberline for grouping.

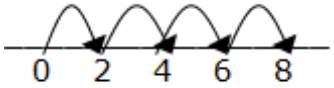
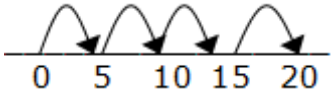
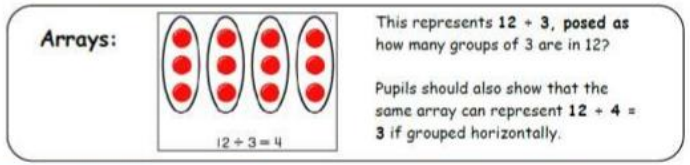
Key vocabulary

share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

Key Skills for division at Year 2

Mental Calculation	Written Calculation
• Count in steps of 2, 3, and 5 from 0 • Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers. • Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts. • Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.	• Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs. • Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Area of division	Concrete	Pictorial	Abstract	Video link
Sharing	6 shared between 2 (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates) 	 This can also be done in a bar so all 4 operations have a similar structure: 	6 ÷ 2 = 3 What's the calculation? 3 | 3	https://vimeo.com/206862682

Grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   Understand division as repeated grouping and subtracting 6 ÷ 2  	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  10 ÷ 5 = ? 5 x ? = 10  	10 ÷ 5 = 2 Divide 10 into 5 groups. How many are in each group?  	https://vimeo.com/206867655
Arrays		 		https://vimeo.com/206870797

Pitch and expectation in Year 2

Children will consolidate their understanding of division as sharing using objects and visual representations. They will then move on to division as grouping using objects such as bead strings. 12 ÷ 3 = 4 Children will move on to recording by drawing arrays. For the example on the left they will start by drawing 3 rows and then keep adding one to each row until they get to 12. The number of columns gives them the answer. 6 ÷ 2 = 3 When the children are confident at using arrays

to group for division, they will move onto using open number lines to do repeated subtraction. Note: Do not work with numbers that have remainders at this stage.

Challenge in Year 2: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 3: Division

Focus: Dividing 2 digit numbers by 1 digit numbers moving from numberline methods to short division Children in year 3 will continue to use a numberline to solve division problems and will begin to jump more than one step at a time in the style of 'chunking'. Once confident they will move on to short division without any remainders.

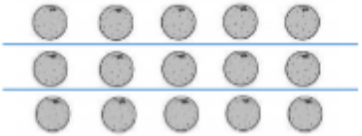
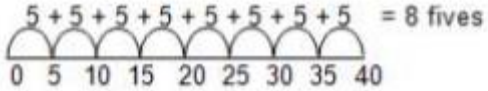
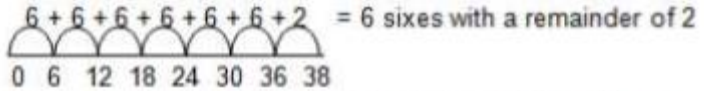
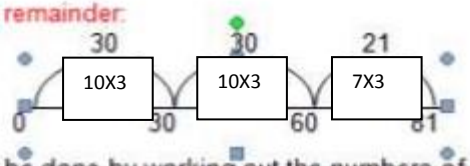
Key vocabulary

share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple

Key Skills for division at Year 3

Mental Calculation	Written Calculation
Recall and use division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables Pupils develop efficient mental methods, for example, using division facts (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3). Halve even numbers up to 50 and multiples of ten to 100 Perform divisions within the tables including those with remainders, e.g. 38 ÷ 5.	Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one digit Solve problems, in contexts, and including missing number problems, involving division Pupils develop reliable written methods for division, starting with calculations of 2digit numbers by 1-digit numbers using a ENL.

Area of division	Concrete	Pictorial	Abstract	Video link

Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 5 x 3 = 15 3 x 5 = 15 15 ÷ 5 = 3 15 ÷ 3 = 5	https://vimeo.com/206870797
Division with numberlines		Example without remainder: 40 ÷ 5 Ask "How many 5s in 40?"  Example with remainder: 38 ÷ 6  For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts. Example without remainder: 81 ÷ 3  This could either be done by working out the numbers of threes in each jump as you go along (10 threes are 30, another 10 threes makes 60, and another 7 threes makes 81. That's 27 threes altogether) or by counting in jumps of known multiples of 3 to reach 81 (30 + 30 + 21) then working out the number of threes in each jump.		

Pitch and expectation in Year 3

Children will begin to use the grouping numberline method to solve problems with remainders. They will start on zero and write the dividend at the end of their numberline. They will jump in steps of the divisor until they get as close to the end as possible. Whatever is left over is the remainder. Using cubes or arrays alongside the numberline will consolidate understanding. Once confident children will begin to solve problems on a grouping numberline involving bigger numbers. To solve this effectively they will need to subtract chunks of the divisor. As you can see in the image for 92 ÷ 4, a step of 10 groups of 4 has been jumped, followed by another step of 10 jumps, and finally followed by a step of 3 jumps of 4. This means that in total 4 was jumped 23 times making 23 the answer. Once children are confident with numberline methods then they should start work on short division. First of all arrays should be used be used to show a division calculation, the same calculation should then be shown in the short multiplication method. Place value should be regularly discussed so children realize that they

are partitioning the dividend and dividing the units then then tens by the divisor.
Please Note: Initially children will start with simple problems where each digit is a multiple of the divisor.

Challenge in Year 3: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 4: Division

Focus: Consolidating and extending use of short division Children in year 4 will continue to use short division to solve division problems. They will begin to work on remainders, including problems where there are remainders in the first numbers but not in the final answer.

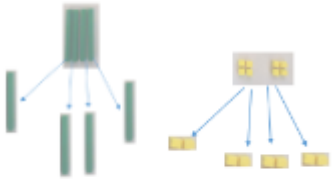
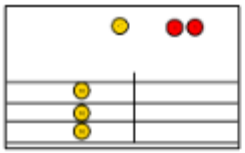
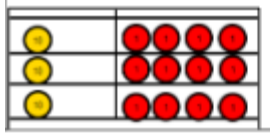
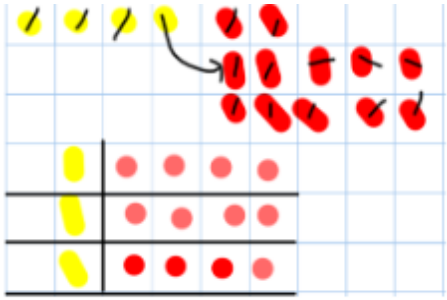
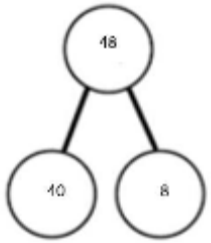
Key vocabulary

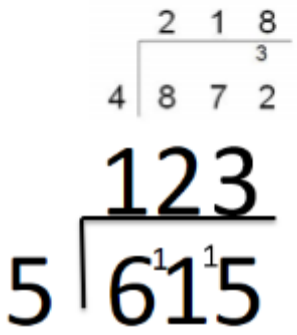
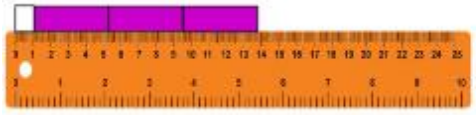
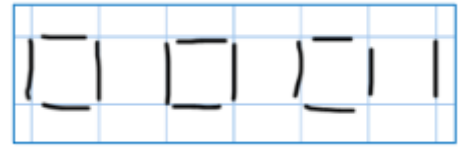
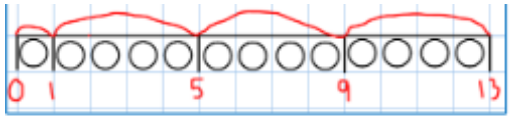
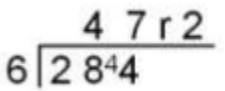
share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor

Key Skills for division at Year 4

Mental Calculation	Written Calculation
Recall multiplication and division facts for all numbers up to 12 x 12. Give remainders as whole numbers. Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1. Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200	Use a written method to divide a 2-digit or a 3-digit number by a single-digit number. Give remainders as whole numbers. Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Area of division	Concrete	Pictorial	Abstract	Video link

Base ten or counters (2 digit divided by 1 digit)	48 ÷ 4 = 12  Start with the tens. 42 ÷ 3= 14  1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10?  Exchange the ten for 10 ones and share out 12 ones 2 digit numbers divided by 1 digit numbers	Children to represent the base 10 and sharing pictorially. 	48 ÷ 4  4 tens ÷ 4 = 1 ten 8 ones ÷ 4 = 2 ones 10 + 2 = 12 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14	

Short division	Use of the 'bus stop method' using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- ***this can also be done using sharing!*** 615 ÷ 5  Step 1: make 615  Step 2: Circle your groups of 5  Step 3: Exchange 1H for 10T and circle groups of 5  Step 4: exchange 1T for 10ones and circles groups of 5	This can easily be represented pictorially, till the children no longer need to do it.	Begin with divisions that divide equally with no remainder. 	https://vimeo.com/206883667
Remainders	2 digit numbers divided by 1 digit with remainders Use of lollipop sticks to form wholes  Use of Cuisenaire rods and rulers (using repeated subtraction) 	 		

Pitch and expectation in Year 4

Once confident with the method of short division, they will move on to problems where the first digit of the dividend is not a multiple of the divisor and

therefore a remainder will need to be carried. Children may need to use other equipment to calculate the division and multiplication facts required. Children who can use short multiplication problems with remainders (but not those in the final answer) are now ready to work on 3 digit problems. Again, there should be remainders in the calculation but never in the final answer.
Once children are confident at dividing with 3 digits, they need to attempt problems where the answer in the first column (hundreds column) is a zero. They may wish to record the hundred initially as this will help them remember its place and the numbers value.

Challenge in Year 4: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 5: Division

Focus: Extending use of short multiplication to 4 digits and remainders Children in year 5 will use short division to solve problems up to 4 digits long. For the first time they will use short division to solve problems that have a remainder in the final answer.

Key vocabulary

share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor quotient, prime number, prime factors, composite number (non-prime)

Key Skills for division at Year 5

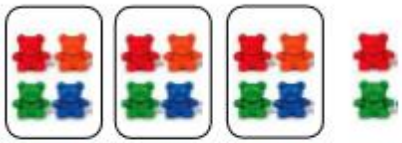
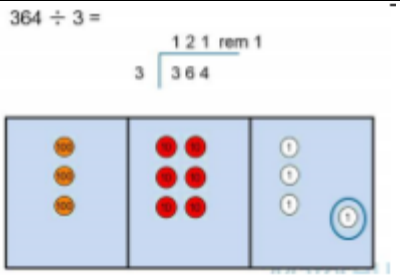
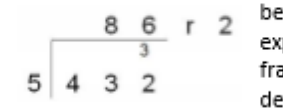
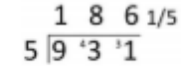
Mental Calculation

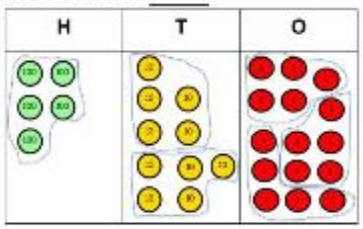
- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19

Written Calculation

- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Use multiplication and division as inverses. Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r 2 = 241/2 = 24.5 ≈ 25).

Area of division	Concrete	Pictorial	Abstract	Video link

Division with remainders (revision)	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. 		Complete written divisions and show the remainder using r.  	
Short division with remainders	364 ÷ 3 =  			Move onto divisions with a remainder. Once children understand remainders, begin to express as a fraction or decimal according to the context.    	https://vimeo.com/206883667

Fluency, reasoning and problem solving		Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?  What's the calculation? What's the answer?  	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group? $5\overline{)615}$ 615 ÷ 5 = ☐ = 615 ÷ 5 How many 5's go into 615?	

Pitch and expectation in Year 5

In year 5 children will begin to solve division problems where a number up to 4 digits is divided by a single digit number including answers with remainders. These division problems need to be contextual so the children learn how to express the remainder- as a number, a fraction, a decimals, rounded up or rounded down.

Challenge in Year 5: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth

Year 6: Division

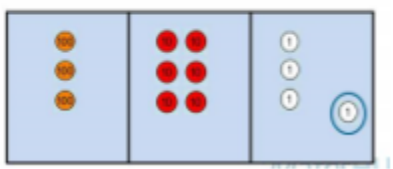
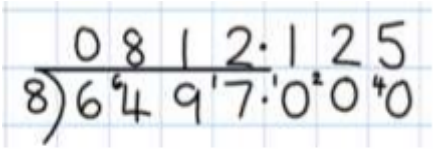
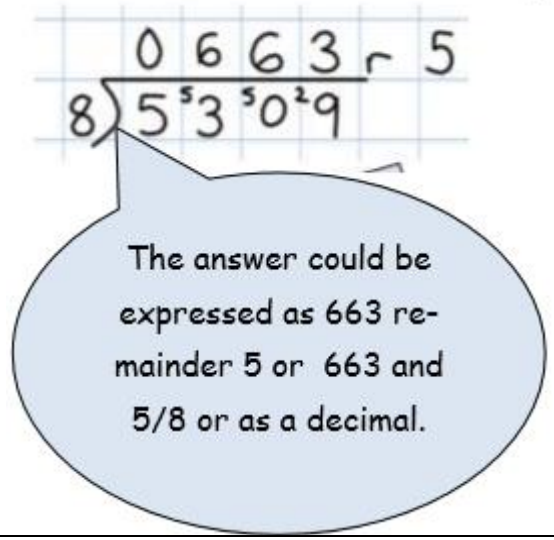
Focus: Using short division to divide 4 digit numbers and express remainders as decimals and long division for dividing 2 digit numbers In year 6, children will use short division to divide decimal numbers by single digit numbers. The final step of division will be long division which will be used to divide numbers by 2 digits.

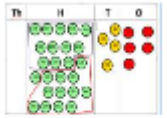
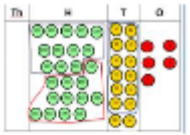
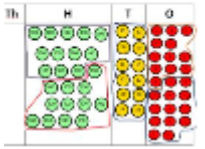
Key vocabulary As previously, & common factor

Key Skills for division at Year 6

Mental Calculation	Written Calculation
• Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations • Perform mental calculations, including with mixed operations and large numbers. • Identify common factors, common multiples and prime numbers. • Use estimation to check answers to calculations and determine accuracy, in the context of a problem.	• Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate. • Solve problems involving all 4 operations. • Use written division methods in cases where the answer has up to two decimal places. • Solve problems which require answers to be rounded to specified degrees of accuracy.

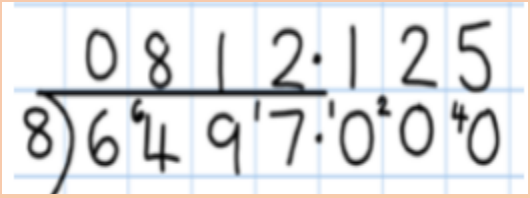
Area of division	Concrete	Pictorial	Abstract	Video link

Short division with remainders in the answer	364 ÷ 3 = 1 2 1 rem 1 3 | 3 6 4 		Children should continue to use short division with remainders. They need to learn how to express an answer as a remainder, a fraction or as a decimal as in in this example. It is important for children to start from real life problem solving contexts and for them to consider how   	https://vimeo.com/206883667

Long division	 0212 12|2544 24 14 12 24 24 0 2544 ÷ 12 How many groups of 12 thousands do we have? None  Exchange 2 thousand for 20 hundreds.  02 12|2544 24 1 How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.  021 12|2544 24 14 12 2 Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.  Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2	Children to represent the counters, pictorially and record the subtractions beneath.	0 12|2544 Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds. 02 12|2544 24 1 Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left. 021 12|2544 24 14 12 2 Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left. 0212 12|2544 24 14 12 24 24 0 Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.	https://vimeo.com/206898235

Pitch and expectation in Year 6

The remainder in this answer would have been 1 but it has been expressed as a decimal.



To do this, children need to insert a decimal point next to the units and carry the remainder over the decimal point. Zeroes are inserted to the right of the

decimal point to show that there was no value.
To divide by 2 digit numbers, the children will use the method of long division. Any remainders would need to be expressed in a way that matched the context of the problem.

Challenge in Year 6: Mastery for greater depth

Mastery assessment document from NCETM provides opportunities for greater depth